

Phenomena such as pain, fervor of belief, or really any subjective measure are notoriously difficult to accurately grasp specifically because they are subjective. Attempts to measure such subjective data points have taken on numerous forms: questionnaires, verbal descriptors, ordinal rating scales and so on. An alternative approach is not to directly measure the subjective phenomenon but rather the perceived change in a subjective phenomenon, e.g. would you rather...?; is this more painful than that?; if you had to choose between x and y?

Fortunately, there is a tidy means of working with such data - the Bradley-Terry model. Since the data are ordinal we are simply looking at the probability of a 1 or a 0 occurring and under what circumstances - which for my fellow statistics nerds you will quickly recognize as leading to a logistic cumulative distribution. Modelling data using the BTM has been used in forecasting of markets, in detecting match fixing by examining the probability of a team losing to another team based on both teams' strengths, or in analyzing the behavior of investors in a market.

Ostensibly we are simply examining  $P_{i=1}/[P_{i=1} + (1-P_{i=1})]$  so the probability that an event occurs divided by the probability that it occurs plus one minus the probability that it occurs (fairly straightforward). Note that the BTM assumes positive quantities so probabilities sum to 1 (note that assumptions such as IIA must be met). If you have read up to this point you may be saying to yourself, "yes but we are simply asking if one stimuli causes something to happen or not. What if there are multiple stimuli, some of which may produce equivalent probabilities?" To address that the BTM (or an extension thereof) has a nice and tidy way of dealing with "ties." To address equivalence (or ties) we simply raise the probability of 1 to the margin between the two phenomena. Think of this as two teams playing each other in a repeated game. Sometimes team A and sometimes team B wins. What if team A and team B each win 50% of the time. In that case we would look at the margin of victory between teams A and B. For team A then let us assume that the margin of victory over B was on average 10 points and for team B over team A it was 5 points. The probability that A is better than B is:

$P(A = 1)^{10}/[P(A = 1)^{10} + P(B = 1)^5]$  so  $\frac{0.50^{10}}{(0.50^{10}+0.50^5)} = 0.03$  which implies a 3% advantage, e.g. 53%

To do this in R there is some fairly tidy code in the BradleyTerry2 package from Turner and Firth, with package documentation here.

I have provided a rough example below.

```
mydata<-read.csv(file = "~/somedata.csv")
```

```
fit1<-BTm(cbind(Y1, Y0), X1, X0, ~X1, id="the name thing you want to classify by",  
data=mydata)
```

#Y1 is the variable for when Y= 1 (such as wins), Y0 is the variable for when Y=0 (such as losses). X1 is the classifier for the thing that is supposed to =1, such as team whose winning

percentage you want to find, and as such X0 would be the opponent). id is a name you assign. To compare multiple models you would simply conduct an ANOVA over multiple models, e.g. fit1, fit2, fit3...